

# Week 3: Problem Solving: The Physics of Bottle Rockets

*Sophomore Clinic Fall 07*

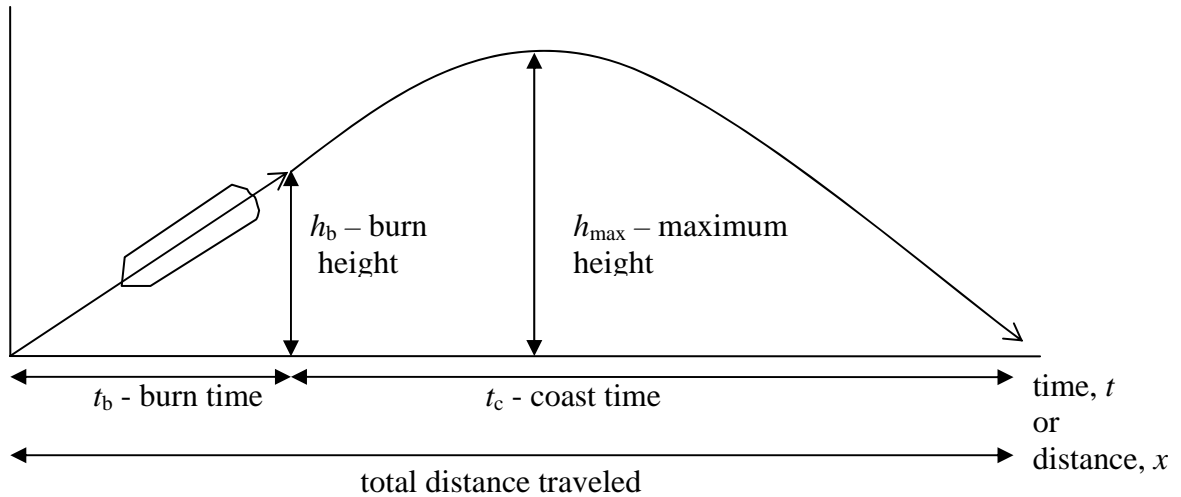
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The assignment this week focuses on the problem solving and analysis components of convergent thinking. This handout will reinforce the basic physics behind rocket propulsion and apply it to the bottle rocket problem. In this handout you will be given design equations that govern part of the problem. You will then learn to perform a further analysis to determine the rocket's trajectory but herein you will be required program your design equations using the physics of ballistic trajectories including drag. Since you will be given most of the setup in the first portion of the problem, the first section could be considered more problem solving than analysis. The latter section could be considered more analysis since you are given less information with which to begin and must program the equations yourself.

## The Basics of Rocket Propulsion

The distance traveled by the bottle rockets can be approximated using concepts from physics; some of the physics you have studied and some you have not. As stated before, the energy for the rocket's motion comes from the stored potential in the pressurized air/water mixture inside the bottle. That energy, specifically the pressure, forces the water out of the rocket at some exit velocity. The mass and velocity of the exiting water have a calculable amount of momentum which is transferred back to the rocket itself. The process is not instantaneous therefore we must define two phases of flight for the rocket.

- During the *burn* phase the rocket's propellant is expelled at some velocity and momentum is transferred to the rocket's remaining mass (note the rocket loses mass as it consumes fuel). As the rocket expels mass it gains velocity.
- During the *coast* phase the rocket is assumed to follow a ballistic trajectory governed by its launch angle relative to the ground and its velocity at the end of the burn phase.



Assumptions:

- Air acts as an ideal gas.
- There is no air drag.
- The air supply tube does not interfere with ejection of water.
- The air inside the rocket remains at a constant pressure during the burn phase.
- The burn time is much smaller than the coast time.

### Problem Solving: Design equations during the burn phase

During the burn phase the velocity of the rocket along its axis is governed by

$$(1) v_f = -g \cos \theta t_b + v_e \ln \frac{M_o}{M_f}$$

which is known as the Rocket Equation. In this equation  $g$  is gravity,  $\theta$  is the angle with respect to the ground,  $t_b$  is the burn time,  $v_e$  is the exit velocity of the water,  $M_o$  is the initial mass of the rocket (water plus bottle with fins plus clay) and  $M_f$  is the final mass of the rocket (bottle, fins, and clay *sans* water) and  $v_f$  is the final velocity of the rocket. In this equation **we know everything except  $t_b$  and  $v_e$ .**

If we assume that  $PV=nRT$  holds for the air inside the rocket and that no energy is lost as the water is expelled, then we can show that:

$$(2) v_e = \left[ \frac{2(P_{bottle} - P_{atmosphere})}{\rho \left(1 - \frac{d_{nozzle}}{d_{bottle}}\right)^4} \right]^{1/2}$$

where pressures are measured in absolute terms inside the bottle and in atmosphere, the  $d$ 's are diameters of a cross-section of the bottle (assume  $5'' = 127$  mm) and the nozzle (assume  $0.85'' = 21.59$ ), and  $\rho$  is the density of water ( $\rho = 0.036$  pound/cubic inch). Therefore in the bottle rockets project for this class  $v_e$  **is fixed**.

With a known value for the velocity of the exiting water, the burn time can be found from

$$(3) t_b = \frac{M_{H20}}{\rho A_{nozzle} v_e}.$$

where  $M_{H20}$  is the initial mass of water.

Given equations (2) and (3) we can determine the initial velocity of the rocket from (1). In reality, the rocket reaches this velocity rather quickly (about 0.2 seconds) **so we can assume that the rocket has the velocity,  $v_f$ , immediately after it leaves the launcher.**

You have performed tests over several of the variables in the design equation, most notably the amount of water, and have hopefully seen how they affect the distance traveled. These equations allow you to perform similar experiments on paper or better yet in a spreadsheet. This type of analysis is often called a *design study*.

### **Analysis: Deriving the ballistic trajectory equations for the rocket**

With a known velocity,  $v_f$ , starting from the origin of the graph (on the ground) and a known launch angle of 45 degrees you can use your knowledge of ballistic trajectories from Physics I to approximate the distance traveled by the rocket.

Hint: The rocket will travel upwards with a component of  $v_f$  in the  $y$ -direction until its velocity reaches zero vertically; this will also yield its maximum height. If you double the time it takes to reach this height, this will approximate  $t_c$  which is in turn a good approximation for the total time the rocket is in the air.

### **Food for Thought:**

1. Do the trends in your design study of distance versus mass of water match your experimental design curves? If so, why do you think this method of analysis is correct? If not, what would you change about the method of analysis?
2. Do the predictions of distance versus mass of water (actual numbers) match your experiments? If not, what factors in the design are not included in the analysis that may account for the difference?
3. Based on the results from the design study (e.g. the analysis you have performed), list design changes/choices that would be appropriate and state specifically how the design study informs those choices/changes..